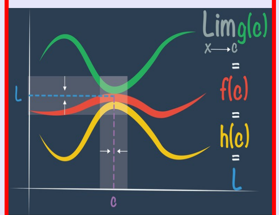
Calculus I

Lecture 10



Feb 19-8:47 AM

Find $\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{x-8} = \dfrac{\sqrt[3]{8}-2}{8-8} = \dfrac{2-2}{8-8} = \dfrac{0}{0}$ I.F.

$A^3-B^3 = (A-B)(A^2+AB+B^2)$

$$\lim_{x\to8}\frac{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{(x-8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}$$

$$=\lim_{x\to8}\frac{\overset{x}{\sqrt[3]{x^3}}+\cancel{2\sqrt[3]{x^2}}+\cancel{4\sqrt[3]{x}}-\cancel{2\sqrt[3]{x^2}}-\cancel{4\sqrt[3]{x}}-8}{(x-8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}$$

$$=\lim_{x\to8}\frac{\cancel{x-8}}{\cancel{(x-8)}(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}=\lim_{x\to8}\frac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}$$

$$=\frac{1}{\sqrt[3]{64}+2\sqrt[3]{8}+4}$$

$$=\boxed{\frac{1}{12}}$$

Sep 11-7:26 AM

Find $\lim\limits_{x\to 8} \dfrac{\sqrt[3]{x}-2}{x-8}$ by making Subs.

Let $u=\sqrt[3]{x} \Rightarrow u^3=x$

as $x\to 8$, $u\to 2$

$$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\lim_{u\to 2}\frac{u-2}{u^3-8}=\frac{2-2}{2^3-8}=\frac{0}{0}\ \text{I.F.}$$

A^3-B^3

$$=\lim_{u\to 2}\frac{\cancel{u-2}}{\cancel{(u-2)}(u^2+2u+4)}$$

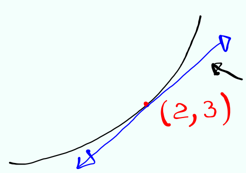
$$=\lim_{u\to 2}\frac{1}{u^2+2u+4}$$

$$=\frac{1}{2^2+2(2)+4}=\boxed{\frac{1}{12}}$$

Sep 11-7:34 AM

Find equation of the tangent line to the graph of $f(x)=x^2-4x+7$ at $x=2$.

$f(2)=2^2-4(2)+7=3$



$$m=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$m=\lim_{h\to 0}\frac{(2+h)^2-4(2+h)+7-3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{4}+4h+h^2\cancel{-8}-4h\cancel{+7}\cancel{-3}}{h}=\lim_{h\to 0}\frac{\cancel{h}(\cancel{4}+h\cancel{-4})}{\cancel{h}}$$

$$=\lim_{h\to 0}h=\boxed{0}$$

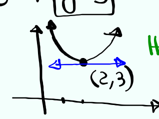
Horizontal line

$y-y_1=m(x-x_1)$

$y-3=0(x-2) \rightarrow y-3=0 \rightarrow \boxed{y=3}$

$f(x)=x^2-4x+7=x^2-4x+4+3$

$f(x)=(x-2)^2+3$

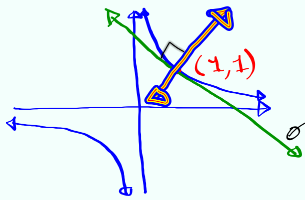


Parabola, opens up, Vertex $(2,3)$

Sep 11-7:38 AM

Find equation of the normal line to the graph of $f(x)=\frac{1}{x}$ at $x=1$.

$f(1)=1$ $\perp$ to tan. line @ tan. Point.



$$m=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{1+h}-1}{h}$$

LCD $=(1+h)$

$$=\lim_{h\to 0}\frac{\cancel{(1+h)}\cdot\frac{1}{\cancel{1+h}}-(1+h)\cdot 1}{h(1+h)}$$

$$=\lim_{h\to 0}\frac{\cancel{1}-\cancel{1}-\overset{-1}{\cancel{h}}}{\cancel{h}(1+h)}$$

$$=\lim_{h\to 0}\frac{-1}{1+h}=\boxed{-1}$$

Slope of tan. line

$$m_{\text{Normal line}}=\frac{-1}{m_{\text{Tan. line}}}=\frac{-1}{-1}=\boxed{1}$$

$y-y_1=m(x-x_1)$

$y-1=1(x-1)\rightarrow\boxed{y=x}$

Normal line@ $(1,1)$

Sep 11-7:49 AM

Prove $\lim_{x\to 2}(5x-3)=7$ $\Rightarrow$ Find a relationship between ε & δ.

$f(x)=5x-3$

$a=2$

$L=7$

For any $\varepsilon>0$, there is a $\delta>0$ such that $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

Verify the limit.

$\lim_{x\to 2}(5x-3)=5(2)-3=7$ ✓

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|5x-3-7|<\varepsilon$ whenever $|x-2|<\delta$

$|5x-10|<\varepsilon$

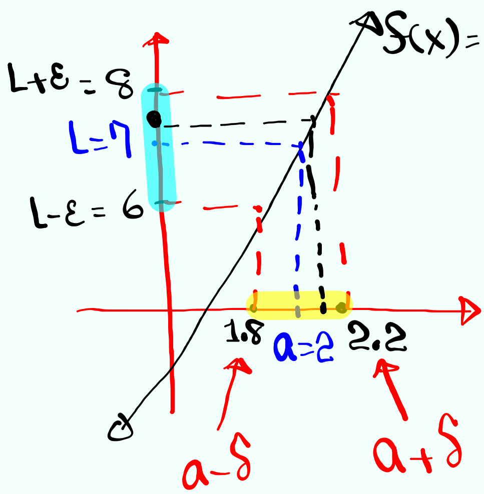
$|5(x-2)|<\varepsilon$

$5|x-2|<\varepsilon$ $\rightarrow$ $|x-2|<\frac{\varepsilon}{5}$

Pick $\delta=\frac{\varepsilon}{5}$

if $\varepsilon=1$, $\delta=\frac{1}{5}=.2$

Sep 11-7:58 AM



$\varepsilon = 1$

$\delta = .2$

Suppose $x = 2.1$

$f(2.1) = 5(2.1) - 3$

$= 10.5 - 3$

$= \boxed{7.5}$

Sep 11-8:04 AM

Prove $\lim_{x \to 3} (|x| + 2) = 5$

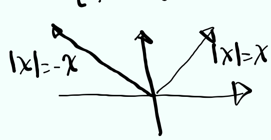
$f(x) = |x| + 2$, $a = 3$, $L = 5$

Verify the limit. $\lim_{x \to 3} (|x| + 2) = |3| + 2 = 5 \checkmark$

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left| |x| + 2 - 5 \right| < \varepsilon$ whenever $|x - 3| < \delta$

$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$



Since $x \to 3$, $|x| = x$

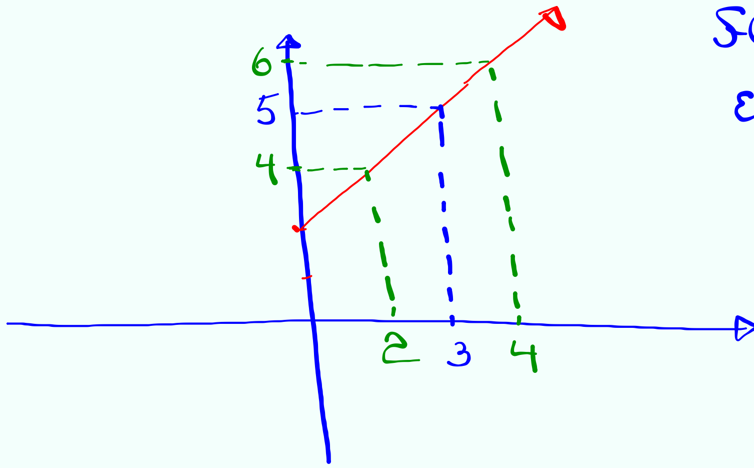
$|x + 2 - 5| < \varepsilon$ whenever $|x - 3| < \delta$

$|x - 3| < \varepsilon$ whenever $|x - 3| < \delta$

Pick $\delta = \varepsilon$

If $\varepsilon = 1$, then $\delta = 1$

Sep 11-8:08 AM



$f(x) = |x| + 2$

$\varepsilon = 1$

Sep 11-8:15 AM

Prove $\lim_{x \to -3} (|x| + 2) = 5$

$f(x) = |x| + 2$

$L = 5$ ✓

$a = -3$

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$||x| + 2 - 5| < \varepsilon$ " $|x - -3| < \delta$

Since $x \to -3$, $|x| = -x$

$|-x + 2 - 5| < \varepsilon$ whenever $|x + 3| < \delta$

$|-x - 3| < \varepsilon$

$|-1(x+3)| < \varepsilon$

$\underbrace{|-1|}_{1} |x+3| < \varepsilon$ → $|x+3| < \varepsilon$

Pick $\delta = \varepsilon$

If $\varepsilon = 2$, then $\delta = 2$

If $\varepsilon = \frac{1}{2}$, then $\delta = \frac{1}{2}$

Sep 11-8:17 AM

Prove $\lim_{x \to 2} x^2 = 4$

$f(x) = x^2$

$L = 4$ ✓

$a = 2$

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 4| < \varepsilon$ " $|x - 2| < \delta$

$|(x+2)(x-2)| < \varepsilon$

$|x+2|\,|x-2| < \varepsilon$

Bound Keep

$5\,|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{5}$

If $\delta \le 1$

$|x-2| \le 1$

$-1 \le x-2 \le 1$

Add 4

$-1+4 \le x-2+4 \le 1+4$

$3 \le x+2 \le 5$

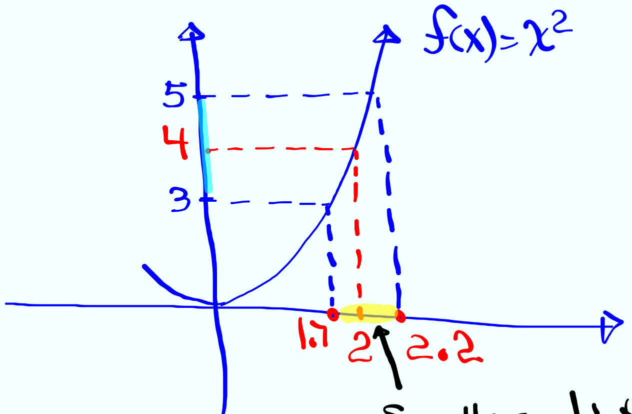
$|x+2| < 5$

$\delta = \min\left\{1, \frac{\varepsilon}{5}\right\}$

$\varepsilon = 1 \rightarrow \delta = \min\left\{1, \frac{1}{5}\right\} = \frac{1}{5}$

$\varepsilon = 10 \rightarrow \delta = \min\left\{1, \frac{10}{5}\right\} = \min\{1, 2\} = 1$

Sep 11-8:23 AM



If $\varepsilon = 1$

$x^2 = 3 \rightarrow x \approx 1.7$

$x^2 = 5 \rightarrow x \approx 2.2$

Smaller distance $= 0.2$

$\delta = \min\left\{1, \frac{\varepsilon}{5}\right\}$

$= \min\left\{1, \frac{1}{5}\right\} = \frac{1}{5} = .2$

Sep 11-8:33 AM